Set theory - Winter semester 2016-17

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Problem 1 (4 Punkte)**.** The *Collection scheme* states that for every relation R and for every set x, there is a set y such that

$$\forall u \in x\ (\exists v\ (u,v) \in R \Rightarrow \exists w \in y\ (u,w) \in R).$$

Prove that the axioms and schemes of ZFC without the Replacement scheme, but with the Collection scheme, imply the Replacement scheme.

Problem 2 (4 points)**.** For this exercise, we assume the axioms and schemes of ZFC without the Foundation scheme, but with the additional axiom

$$\forall x\ \exists y\ (x \in y \wedge \mathrm{Trans}(y)).$$

(1) Prove that the *Foundation Axiom*

$$\forall x\ (x \neq \emptyset \Rightarrow \exists y \in x\ x \cap y = \emptyset)$$

implies the Foundation scheme.

(2) Prove that for every set x, there is a $\subseteq$-minimal transitive set y with $x \in y$.

Problem 3 (6 points)**.** Prove the following statements.

(1) If x is a transitive set, then $x = \emptyset$ or $\emptyset \in x$.
(2) If x is a transitive set, then $\bigcup x$ is a transitive set.
(3) If A is a class term and A is transitive, then $\bigcap A$ is a transitive set.
(4) There is a transitive set that is not an ordinal.
(5) If x is a set of ordinals, then $\sup(x) := \bigcup x$ is an ordinal.
(6) If $x \in y$ and y is an ordinal, then x is an ordinal.

Problem 4 (6 points)**.** Prove that a set x is an ordinal if and only if x is transitive and $(x, \in)$ is a strict linear order.

Due Friday, November 04, before the lecture.